

**EV316936591**

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

APPLICATION FOR LETTERS PATENT

**Methods and Systems for Fingerprinting Digital Data**

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ATTORNEY'S DOCKET NO. MS1-393USC1

1     **RELATED APPLICATIONS**

2             This application is a continuation of and claims priority to U.S. Patent  
3     Application Serial No. 09/437,713, filed on October 28, 1999, the disclosure of  
4     which is incorporated by reference herein.

5  
6     **TECHNICAL FIELD**

7             This invention pertains to methods and systems for fingerprinting digital  
8     data.

9  
10    **BACKGROUND**

11            Fingerprinting is a technique that involves uniquely marking each copy of a  
12    particular object, and associating each uniquely marked copy with a particular  
13    entity to which the copy is distributed. If unauthorized copies of the uniquely  
14    marked copy are made, the fingerprint can be traced back to the original entity to  
15    which the copy was initially distributed.

16            As an example, consider a printed map. When a map maker produces a  
17    map, they may want to ensure that those individuals to whom the map is  
18    distributed do not make unauthorized copies of the map and distribute them to  
19    others. One way that the map maker might protect his maps is to introduce a  
20    different trivial error, or fingerprint, (e.g. a non-existent street) into each of the  
21    copies of the map that are distributed. Each fingerprint is then associated with an  
22    individual to whom the map is to be distributed. By associating each different  
23    fingerprint with a different individual, if and when unauthorized copies of that  
24    individual's copy are uncovered, they can be traced back to the original individual  
25    by virtue of the unique fingerprint that the map contains.

1 One problem with this type of fingerprinting can arise when two or more  
2 individuals collude for the purpose of discovering their fingerprints. That is, when  
3 two or more individuals get together and compare their maps, they can, given  
4 enough time, ascertain their unique fingerprints by simply looking for the  
5 differences between their maps. If they can ascertain their fingerprint, they can  
6 alter it and therefore possibly avoid detection.

7 In contemporary times, particularly with the advent of the Internet and  
8 electronic distribution, fingerprinting digital data (e.g. software, documents,  
9 music, and video) for purposes of detecting or deterring unauthorized copying has  
10 become particularly important. As in the above map example, collusion by  
11 different individuals in the digital context can pose challenges to the owners and  
12 distributors of such digital data. Although progress has been made in the area of  
13 digital fingerprinting, further strides are necessary to increase the breadth of  
14 protection that is afforded by digital fingerprinting. For example, in one  
15 fingerprinting system (the “Boneh-Shaw system” discussed in more detail below),  
16 some protection against collusion is provided, but only when the number of  
17 colluders is relatively small. Thus, there is a need to increase the protection that is  
18 provided by digital fingerprinting to provide detection of colluders even when the  
19 number of colluders is large.

20 Accordingly, this invention arose out of concerns associated with providing  
21 improved methods and systems for fingerprinting digital data.

## 22 23 SUMMARY

24 Methods and systems for fingerprinting digital data are described. In the  
25 described embodiment, Direct Sequence Spread Spectrum (DSSS) technology is

utilized. Unique fingerprinting words are defined where each includes at least one spread sequence. In the described embodiment, a fingerprinting word comprises a plurality symbols, called " $\Gamma$  symbols." Each  $\Gamma$  symbol is composed of  $2c-1$  blocks, where  $c$  represents the number of colluders that are desired to be protected against. Each block contains  $d$  spread sequence chips. The fingerprinting words are assigned to a plurality of entities to which protected objects embedded with the fingerprinting words are to be distributed.

To ascertain the identity of an entity that has altered its unique fingerprinting word, the relative weight of each block is computed in accordance with a defined function and blocks whose weights satisfy a predetermined relationship are "clipped" to a so-called working range. Each  $\Gamma$ -symbol of the altered fingerprinting word is then processed to produce a set of one or more "colors" that might be the subject of a collusion. Each  $\Gamma$ -symbol in the fingerprinting word for each entity is then evaluated against a corresponding produced set and the entity having the most overall incriminating "colors" is incriminated.

## **BRIEF DESCRIPTION OF THE DRAWINGS**

Fig. 1 is a diagram of a computer system that can be utilized in connection with various aspects of the invention.

Fig. 2 is a table that contains a plurality of values that are assignable to various users in connection with the Boneh-Shaw system.

Fig. 3 is a table that contains a plurality of values that are assignable to various users in connection with the described embodiment.

1 Fig. 4 is a flow diagram that describes steps in an embedding method in  
2 accordance with the described embodiment.

3 Fig. 5 is a flow diagram that describes steps in a detection method in  
4 accordance with the described embodiment.

5 Fig. 6 is a flow diagram that describes steps in a detection method in  
6 accordance with the described embodiment.

## 7 8 **DETAILED DESCRIPTION**

### 9 **Overview**

10 In the described embodiment, digital data or objects are fingerprinted, i.e.  
11 embedded, with unique fingerprinting words. Each fingerprinting word is  
12 associated with one of a number of entities or users to which the fingerprinted  
13 objects are to be distributed. In the described scheme, each fingerprinting word  
14 contains a plurality of  $\Gamma$ -symbols, and each  $\Gamma$ -symbol contains a plurality of  
15 blocks. Each block, in turn, comprises a spread sequence that has a plurality of  
16 spread sequence chips.

17 When an altered object is received, it is first processed to identify the  
18 embedded spread sequence chips. Once the chips are identified, a relative weight  
19 function is defined and used to calculate the relative weight for each block. The  
20 relative weight calculations for each block are analyzed in accordance with a  
21 predetermined relationship which determines which of the blocks gets “clipped” to  
22 a predefined working range. The clipped blocks are those that are likely to be  
23 “unseen” in the sense that the colluders who colluded to produce the altered object  
24 likely were not able to see these blocks, i.e. they were the same. The blocks that  
25

are not clipped constitute those blocks that likely were “seen” and therefore possibly altered by the colluders.

With the relative weights of each block having been computed, and the working range defined, each  $\Gamma$ -symbol of the altered object is processed to produce a set of possible  $\Gamma$ -symbols that might be the subject of a collusion. The collection of sets defines a matrix. Each  $\Gamma$  symbol for a user’s unique fingerprint is then compared with the set for each corresponding  $\Gamma$ -symbol in the matrix and a count is kept of the number of times each user’s  $\Gamma$  symbol coincides with a  $\Gamma$ -symbol that is found in a particular set. When all of the users have been thus evaluated, the user with the highest count is selected as a colluder that produced the altered object.

### **Exemplary Computer System**

Fig. 1 shows a general example of a computer 130 that can be used in accordance with the invention. Various numbers of computers such as that shown can be used in the context of a distributed computing environment.

Computer 130 includes one or more processors or processing units 132, a system memory 134, and a bus 136 that couples various system components including the system memory 134 to processors 132. The bus 136 represents one or more of any of several types of bus structures, including a memory bus or memory controller, a peripheral bus, an accelerated graphics port, and a processor or local bus using any of a variety of bus architectures. The system memory 134 includes read only memory (ROM) 138 and random access memory (RAM) 140. A basic input/output system (BIOS) 142, containing the basic routines that help to

1 transfer information between elements within computer 130, such as during start-  
2 up, is stored in ROM 138.

3 Computer 130 further includes a hard disk drive 144 for reading from and  
4 writing to a hard disk (not shown), a magnetic disk drive 146 for reading from and  
5 writing to a removable magnetic disk 148, and an optical disk drive 150 for  
6 reading from or writing to a removable optical disk 152 such as a CD ROM or  
7 other optical media. The hard disk drive 144, magnetic disk drive 146, and optical  
8 disk drive 150 are connected to the bus 136 by an SCSI interface 154 or some  
9 other appropriate interface. The drives and their associated computer-readable  
10 media provide nonvolatile storage of computer-readable instructions, data  
11 structures, program modules and other data for computer 130. Although the  
12 exemplary environment described herein employs a hard disk, a removable  
13 magnetic disk 148 and a removable optical disk 152, it should be appreciated by  
14 those skilled in the art that other types of computer-readable media which can  
15 store data that is accessible by a computer, such as magnetic cassettes, flash  
16 memory cards, digital video disks, random access memories (RAMs), read only  
17 memories (ROMs), and the like, may also be used in the exemplary operating  
18 environment.

19 A number of program modules may be stored on the hard disk 144,  
20 magnetic disk 148, optical disk 152, ROM 138, or RAM 140, including an  
21 operating system 158, one or more application programs 160, other program  
22 modules 162, and program data 164. A user may enter commands and  
23 information into computer 130 through input devices such as a keyboard 166 and a  
24 pointing device 168. Other input devices (not shown) may include a microphone,  
25 joystick, game pad, satellite dish, scanner, or the like. These and other input

1 devices are connected to the processing unit 132 through an interface 170 that is  
2 coupled to the bus 136. A monitor 172 or other type of display device is also  
3 connected to the bus 136 via an interface, such as a video adapter 174. In addition  
4 to the monitor, personal computers typically include other peripheral output  
5 devices (not shown) such as speakers and printers.

6 Computer 130 commonly operates in a networked environment using  
7 logical connections to one or more remote computers, such as a remote computer  
8 176. The remote computer 176 may be another personal computer, a server, a  
9 router, a network PC, a peer device or other common network node, and typically  
10 includes many or all of the elements described above relative to computer 130,  
11 although only a memory storage device 178 has been illustrated in Fig. 1. The  
12 logical connections depicted in Fig. 1 include a local area network (LAN) 180 and  
13 a wide area network (WAN) 182. Such networking environments are  
14 commonplace in offices, enterprise-wide computer networks, intranets, and the  
15 Internet.

16 When used in a LAN networking environment, computer 130 is connected  
17 to the local network 180 through a network interface or adapter 184. When used  
18 in a WAN networking environment, computer 130 typically includes a modem 186  
19 or other means for establishing communications over the wide area network 182,  
20 such as the Internet. The modem 186, which may be internal or external, is  
21 connected to the bus 136 via a serial port interface 156. In a networked  
22 environment, program modules depicted relative to the personal computer 130, or  
23 portions thereof, may be stored in the remote memory storage device. It will be  
24 appreciated that the network connections shown are exemplary and other means of  
25 establishing a communications link between the computers may be used.



1 Generally, the data processors of computer 130 are programmed by means  
2 of instructions stored at different times in the various computer-readable storage  
3 media of the computer. Programs and operating systems are typically distributed,  
4 for example, on floppy disks or CD-ROMs. From there, they are installed or  
5 loaded into the secondary memory of a computer. At execution, they are loaded at  
6 least partially into the computer's primary electronic memory. The invention  
7 described herein includes these and other various types of computer-readable  
8 storage media when such media contain instructions or programs for implementing  
9 the steps described below in conjunction with a microprocessor or other data  
10 processor. The invention also includes the computer itself when programmed  
11 according to the methods and techniques described below.

12 For purposes of illustration, programs and other executable program  
13 components such as the operating system are illustrated herein as discrete blocks,  
14 although it is recognized that such programs and components reside at various  
15 times in different storage components of the computer, and are executed by the  
16 data processor(s) of the computer.

### 17 18 **The Boneh-Shaw System**

19 The Boneh-Shaw system (hereinafter "the BS-system") is a fingerprinting  
20 system for use with digital data. The BS-system attempts to overcome the  
21 problem of collusion when fingerprinting digital data. Aspects of the B-S system  
22 are described in an article entitled "Collusion-Secure Fingerprinting for Digital  
23 Data" authored by Boneh and Shaw, appearing in IEEE Transactions on  
24 Information Theory, Vol. 44, No. 5, September 1998.

1           One of the principle assumptions in the B-S system is known as the  
2 “marking assumption”: that users cannot alter marks if they cannot determine  
3 which data comprise the marks. When an object is fingerprinted, it is embedded  
4 with a fingerprinting word that is unique for each entity or user. By colluding,  
5 users can detect a specific mark if it differs between their copies; otherwise, a  
6 mark cannot be detected. This is the basis of the marking assumption—that is,  
7 users cannot change marks that they cannot see. These marks are referred to as  
8 “unseen” marks.

9           In the B-S system, each user is assigned a unique fingerprinting word. An  
10 example of fingerprinting word assignments is shown in Fig. 2 for five users.  
11 Each row corresponds to a user and shows blocks that form the fingerprinting  
12 word for that user. For example, user 1 has a fingerprinting word  
13 “1111111111111111”, user 2 has a fingerprinting word “0000111111111111”, and  
14 so on for each of the users. The collection of the fingerprinting words for all of  
15 the users defines a step structure that is illustrated by the bold line through the  
16 table. This stepped structure is instrumental in ascertaining potential colluders as  
17 will become apparent below.

18           Each fingerprinting word is divided into a number of blocks that, in turn,  
19 include a plurality of bits. In this example, there are four blocks that are  
20 designated as block 0, block 1, block 2, and block 3. Each of the blocks includes,  
21 in this example, four bits. For purposes of this discussion, the matrix that is  
22 defined by the fingerprinting word assignments is known as a “ $\Gamma$ -code”. As there  
23 can be many, many users, the  $\Gamma$ -code necessary to provide fingerprinting words for  
24 all of the users will be quite large.  
25

1 In accordance with the B-S system, a single permutation of the columns of  
2 the  $\Gamma$ -code is performed before embedding an object with a fingerprint word. An  
3 exemplary permutation is shown in Table 1 below where the order of the blocks is  
4 changed. For simplicity, the permutation as represented in the table above occurs  
5 over whole blocks. In reality, the permutation occurs at the bit level. For  
6 example, the column of leftmost bits might be moved to bit position 12. This  
7 permutation is uniform for all of the users and is known only to the encoder or  
8 embedder and the decoder:

User	Block 2	Block 1	Block 3	Block 0
1	1111	1111	1111	1111
2	1111	1111	1111	0000
3	1111	0000	1111	0000
4	0000	0000	1111	0000
5	0000	0000	0000	0000

16 Table 1

17  
18 When an object is fingerprinted, it is embedded with a permuted  
19 fingerprinting word that corresponds to one of the users. For purposes of  
20 discussion, an “object” is any digital data that is suitable for fingerprinting.  
21 Examples of such objects include, without limitation, documents, music, and  
22 video. When an illegal copy of a protected object is made, a user will typically  
23 attempt to alter their fingerprinting word so as to avoid detection. The BS-system  
24 is directed to ascertaining, with a desirable degree of certainty, the identity of one  
25

1 or more users that may have collaborated in the altering of a protected object.  
2 This is done by examining the altered object.

3 In the discussion that follows, the altered object is represented as  $x$  where  $x$   
4 is a binary word of length  $u$ , and  $I = \{i_1 \dots i_r\}$  is a subset of bit locations of  $x$ , i.e.  $I \subseteq$   
5  $\{1 \dots n\}$ . The notation  $x \downarrow I$  denotes the restriction of word  $x$  to the bit locations of  $I$ .  
6 Let  $W(x)$  denote the Hamming weight of the string  $x$ . The Hamming weight of a  
7 binary string of 1's and 0's is the number of 1's in the string. Likewise, if the  
8 string is composed of +1's and -1's, we could define it to be the number of +1's in  
9 the string.

### 11 The First Algorithm

12 The BS-system employs a first algorithm that is directed to finding a subset  
13 of a coalition that produced an altered object  $x$ . Thus, at this point, an altered  
14 object has been produced by two or more users and an attempt is going to be made  
15 to identify a subset of users that likely produced the object  $x$ . Before describing  
16 the algorithm that produces a subset of likely user candidates, consider the  
17 following. When an altered object  $x$  is received, it will inevitably contain some  
18 form of a fingerprinting word. Recall that each user is assigned a unique permuted  
19 fingerprinting word, an example of which is given in Table 1 above. Because each  
20 user is assigned a unique fingerprinting word, certain aspects of the fingerprinting  
21 word will be unique to each user. For example, a unique aspect of user 1's  
22 fingerprinting word in Fig. 2 is that block 0 comprises all 1's. Each of the other  
23 users has all 0's in their corresponding block 0. Thus, if users other than user 1 are  
24 colluders, then, in accordance with the marking assumption (which states that  
25 users cannot modify "unseen" bits), none of the bits in block 0 will be modified.

1 Accordingly, all of the bits in block 0 will be 0 and user 1 can be ruled out as a  
2 colluder. On the other hand, if any of the bits in block 0 of the altered object  $x$  are  
3 determined to be 1, then user 1 can be incriminated as a colluder. Again, this is  
4 because the bits of block 0 are only capable of being “seen” by a collusion that  
5 includes user 1 because they are different from the bits in block 0 for all of the  
6 other users. Thus, the first algorithm simply looks at the fingerprinting word in  
7 the altered object and attempts to identify, with a desired degree of certainty,  
8 which users are possible candidates for incrimination given that certain bits or  
9 blocks have been modified. It does this by considering the Hamming weight of  
10 particular blocks that are or can be uniquely seen by particular users.

11 As a more concrete example, consider that users 3 and 4 are going to  
12 collude to change a fingerprinting word on their protected objects. Users 3 and 4  
13 will thus compare their permuted fingerprinting words. From Table 1 above, this  
14 comparison will be as follows:

15  
16

User	Block 2	Block 1	Block 3	Block 0
3	1111	0000	1111	0000
4	0000	0000	1111	0000

17  
18  
19  
20

21 When users 3 and 4 compare their fingerprinting words, the bits that appear  
22 in blocks 1, 3, and 0 are “unseen” to the users. This is because they contain the  
23 same values. Thus, in accordance with the marking assumption, the users cannot  
24 change the values of any of the bits at these locations. The bits that appear in  
25 block 2, however, are different as between the users, i.e. they are “seen”.

Accordingly, users 2 and 3 will recognize that because of this difference, there must be a fingerprint in block 2. Knowing this, they can then modify the fingerprint of block 2 so as to avoid detection. In this example, the resulting fingerprinting words might look like this:

User	Block 2	Block 1	Block 3	Block 0
3	0011	0000	1111	0000
4	0011	0000	1111	0000

Here, they changed the first two bits in block 2 from “1” to “0”. Note that they would not change all of the bits of block 2 because then the resultant fingerprinting word would be that of user 4 and would result in user 4’s incrimination as a colluder. When the blocks are unpermuted, the resulting  $\Gamma$ -code looks like this:

User	Block 0	Block 1	Block 2	Block 3
1	1111	1111	1111	1111
2	0000	1111	1111	1111
3	0000	0000	0011	1111
4	0000	0000	0011	1111
5	0000	0000	0000	0000

One thing that the reader will notice is that there is still some semblance of a step function that is defined for user 3 by blocks 1 and 2. This step function, as

1 was pointed out above, is unique for user 3 at the location of blocks 1 and 2. That  
2 is, all of the other users, either above or below user 3 have, respectively, all 1's or  
3 all 0's in their blocks 1 and 2.

4 What the first algorithm does is that, after the columns are unpermuted, it  
5 looks for this unique step function or some semblance thereof for users other than  
6 the first and last users. For the first and last users, the algorithm simply looks for  
7 the unique bits in the blocks that are unique for the first and last users. When a  
8 step function (or unique bits) are located, a corresponding user can be  
9 incriminated. In this example, since the step function still exists for user 3, user 3  
10 can be incriminated. This can be mathematically represented as follows ( $\epsilon$  is the  
11 incrimination error probability):

### 12 **Algorithm 1**

- 13 1. If  $W(x \downarrow \text{Block } 1) > 0$ , then user 1 is incriminated.
- 14 2. If  $W(x \downarrow \text{Block } (n-1)) < d$ , then user  $n$  is incriminated.
- 15 3. For all  $s=2$  to  $n-1$  do:  
16     Let  $R_s = (B_{s-1} \cup B_s)$  (i.e. the bit locations of those two adjacent blocks.)  
17     Let  $K = W(x \downarrow R_s)$ .  
18     If  $W(x \downarrow \text{Block } (s-1)) < K/2 - ((K/2)\log(2n/\epsilon))^{1/2}$ , then user "s" is  
19     incriminated.

### 20 **The Second Algorithm**

21 As was pointed out above, the number of potential users of a given  
22 protected object can be quite large. Thus, using the  $\Gamma$ -code approach discussed  
23 above will, accordingly, result in fingerprinting words that are very large in size.  
24 The second algorithm of the BS-system is directed to incriminating a user or  
25 colluder without having to use such a large  $\Gamma$ -code. When using this algorithm, let

$c$  represent the number of colluders that are desired to be defended against. A  $\Gamma$ -code is then selected to have  $2c$  rows. In this system each row is also referred to as a “color”. So, for example, if one wants to defend against 20 colluders, then a  $\Gamma$ -code is selected that has 40 rows or colors. Each row or color in the  $\Gamma$ -code comprises a plurality of blocks that make up a  $\Gamma$ -symbol. Each color or  $\Gamma$ -symbol is treated as a letter in an alphabet that is defined by the  $\Gamma$ -code. The letters in the alphabet are then used to build unique fingerprinting words for each of the users of the protected object. That is, fingerprinting words contain  $L$  colors or  $\Gamma$ -symbols, where  $L$  is a number that is selected to be large enough so that, given the number of users that are to be assigned fingerprinting words, each is assured of being assigned a unique fingerprinting word.

As an example, consider the following. Assume that it is desirable to defend against 3 colluders at any given time. Thus, a  $\Gamma$ -code is defined to have  $2(3)=6$  colors or  $\Gamma$ -symbols. This is illustrated in the Table 2 below:

Color	$\Gamma$ symbol
1	$\Gamma_1$
2	$\Gamma_2$
3	$\Gamma_3$
4	$\Gamma_4$
5	$\Gamma_5$
6	$\Gamma_6$

Table 2



Consider further, in this example, that in the universe of users, the number of  $\Gamma$  symbols that are necessary for each user to be assigned a unique fingerprinting word is 3, that is  $L=3$ . So, user 1 might be assigned a fingerprinting word ( $\Gamma_4 \Gamma_5 \Gamma_3$ ), user 2 might be assigned a unique fingerprinting word ( $\Gamma_3 \Gamma_5 \Gamma_2$ ), and so on for all of the users. Each of the protected objects are embedded with a permuted form of one of the fingerprinting words. Now, when an altered object is found, applying the principles of Algorithm 1 to each of the  $\Gamma$  symbols in the altered object will yield a set of colors or  $\Gamma$ -symbols that are likely the subject of a collusion. So, in this example, there are three  $\Gamma$  symbols that comprise the altered fingerprinting word. Algorithm 1 is applied to each of the three  $\Gamma$  symbols. The result of this computation yields a set of colors or  $\Gamma$ -symbols for each  $\Gamma$  symbol of the altered fingerprinting word. So, for the first  $\Gamma$  symbol of the altered fingerprinting word, the set of colors (1, 2, 3), i.e.  $\Gamma_1 \Gamma_2 \Gamma_3$ , might be produced. For the second  $\Gamma$  symbol of the altered fingerprinting word, the set of colors (2, 4), i.e.  $\Gamma_2 \Gamma_4$ , might be produced. For the third  $\Gamma$  symbol of the altered fingerprinting word, the set of colors (3, 6), i.e.  $\Gamma_3 \Gamma_6$ , might be produced. These results are summarized in the table below:

$\Gamma$ symbol	Color Set
First $\Gamma$ symbol	1, 2, 3
Second $\Gamma$ symbol	2, 4
Third $\Gamma$ symbol	3, 6

From the collection of possible color sets, the BS-system builds a word or vector by selecting, at random, one and only one color from each color set. In this

1 example, a word might be built by selecting color 1 from the color set associated  
2 with the first  $\Gamma$  symbol, color 4 from the color set associated with the second  $\Gamma$   
3 symbol, and color 6 from the color set associated with the third  $\Gamma$  symbol. Thus,  
4 the word that is built is as follows:  $\Gamma_1 \Gamma_4 \Gamma_6$ . Now, the user having a fingerprinting  
5 word that is closest to this word is incriminated. More detailed information on the  
6 BS-system and its proofs can be found in the article referenced above. Algorithm  
7 2 is summarized just below.

## 8 9 **Algorithm 2**

- 10  
11 1. Apply Algorithm 1 to each of the  $L$   $\Gamma$ -symbols. For each of the  $L$   
12 components arbitrarily choose one of the outputs of Algorithm 1.  
13 Set  $y_i$  to be that chosen output ( $y_i$  is an integer in  $[1,n]$ ). Form the  
14 word  $y = (y_1 \dots y_L)$ .
- 15 2. Find the fingerprinting word that is closest to  $y$ , and incriminate the  
16 corresponding user or entity.

17  
18 In the BS-system, the length in bits of the fingerprinting word or sequence  
19 is given by the following equation:  $O(c^4 \log(N/\epsilon) \log(1/\epsilon))$ , where “ $c$ ” is the size of  
20 the collusion, “ $N$ ” is the number of users, and  $\epsilon$  is the incrimination error  
21 probability. Suppose that it is desirable to protect a 2-hour long object in a system  
22 that is able to robustly hide 1 bit/sec. The number of colluders that can be  
23 protected against, assuming the  $N=10^6$ , and  $\epsilon=10^{-3}$  is just  $c=4$ . Protecting against  
24 just four colluders, while a step in the right direction, does not go far enough for  
25 defending against the possibility that larger numbers of users might get together  
and collude.

## **Inventive Methods and System Overview**

In accordance with the inventive methods and systems, aspects of the BS-system are exploited in conjunction with the use of spread spectrum technology. A spread spectrum sequence is associated with individual blocks of individual fingerprint words. The spread spectrum sequence utilizes a data structure called a “chip” that is embedded in the protected object. The use of spread sequences in the embedding process enables redefinition of the relative weight of each block as well as redefinition of a working range (defined below). The new weights and working range are utilized in connection with an analysis that increases the robustness of the protectiveness over that of conventional methods and systems provide.

## **Spread Spectrum**

Before discussing the details of the inventive methods and systems, some basic background information on spread spectrum technology is given. For additional background on spread spectrum technology, the reader is referred to a text entitled “Spread Spectrum Communications Handbook” Revised Edition (1994), authored by Simon, Omura, Scholtz, and Levitt.

An object that is desired to be protected can be represented as a vector  $m=(m_1,...m_u)$ . This vector can represent pixels in a movie or any type of suitable digital content that is desirable to protect. The components of this vector are viewed over some large alphabet size, e.g.  $m_1$  could be an 8-bit byte that can have a value from between  $-128$  to  $+128$ . Spread spectrum chips  $x = (x_1,...x_u)$  are utilized that have values that are measured in the same units as the individual components of the protected object vector, but which have values that are small in

1 comparison to the values that the individual vector components can have, e.g. the  
2 chips have values that are in  $\{+1, -1\}$ . That is, values of  $x$  are selected to be small  
3 enough that when they are added to  $m$  they are difficult if not impossible to detect.

4 A spread sequence can be utilized to embed data symbols that are in  $\{+1, -$   
5  $1\}$ . These embedded data symbols are different from the individual values  $\{+1, -$   
6  $1\}$  that a spread spectrum chip can have, and therefore the notation  $\{+D, -D\}$  is  
7 utilized to represent the data symbols  $\{+1, -1\}$  so as to avoid confusion. When a  
8 data symbol  $+D$  or  $-D$  is to be embedded, the vector  $m$  for the object is combined  
9 with the appropriate spread spectrum chips. To embed a  $+D$  we add the spread  
10 sequence as is, while to embed  $-D$  we flip the chips (i.e. take the 1s complement  
11 of the sequence) of the spread sequence before adding it. So, to embed  $+D$  we  
12 compute a new vector  $b$  as follows:  $(\forall j)[b_j = m_j + x_j]$ , and to embed  $-D$  we compute  
13  $(\forall j)[b_j = m_j - x_j]$ . When such an embedded object is to be detected, the vector  $b$   
14 can be multiplied by the vector  $x$  and summed over all of the vector components.  
15 The summing of the resultant vector components will indicate whether a data  
16 symbol  $+D$  or  $-D$  was embedded, as will be understood by those skilled in the art.

### 17 18 Embedding

19 In the discussion that follows, four specific types of data structures are  
20 defined and used in the embedding/detection process, i.e. chips, blocks,  $\Gamma$ -symbols  
21 and fingerprinting words. While the latter three data structures share the same  
22 *names* as those discussed above in connection with the BS-system, their  
23 definitions render them completely different and represent a significant departure  
24 from the BS-system, as will become apparent below.

1 A “chip” is the smallest of the data structures and refers to a spread  
 2 spectrum chip. Spread spectrum chips are designated as  $x = (x_1, \dots, x_u)$  and have  
 3 values in  $\{+1, -1\}$ . As in the above discussion on spread spectrum technology, the  
 4 data symbols that are embedded through the use of the spread spectrum chips are  
 5 in  $\{+D, -D\}$ . A “block” is composed of  $d$  chips, where  $d$  represents a parameter  
 6 that controls the error rate. The blocks are designated as  $C_1 \dots C_k$ , where an  
 7 individual block  $i$  is defined as  $C_i = (c_{i1} \dots c_{id})$ , with  $c_{i1} \dots c_{id}$  constituting the  
 8 individual spread spectrum chips. The 1s complement of block  $C_i$  is denoted  $C'_i$ .  
 9 A “ $\Gamma$ -symbol” comprises a plurality of blocks. In the described embodiment, a  $\Gamma$ -  
 10 symbol is composed of  $2c-1$  blocks, where  $c$  represents the number of colluders  
 11 that are desired to be defended against. Last of the data structures is the  
 12 fingerprinting *word* which is composed of  $L$   $\Gamma$ -symbols, where  $L$  represents a  
 13 particular number that is selected to ensure that all of the users in the relevant user  
 14 universe receive unique fingerprinting words.

15 Each user is first assigned a unique fingerprinting word. In the described  
 16 embodiment, the fingerprinting words incorporate a spread sequence rather than  
 17 the individual bits as in the BS-system. Specifically, in the described embodiment,  
 18 each block  $B_i$  of the  $\Gamma$  code in the BS-system is replaced with a suitable spread  
 19 sequence. In this example, blocks that are supposed to be a  $1^d$  in the BS-system  
 20 are replaced with  $C_i$ , and blocks that are supposed to be  $0^d$  are replaced with the 1s  
 21 complement  $C'_i$ . An exemplary  $\Gamma$  code in accordance with this embodiment is  
 22 shown in Fig. 3. Once the users have been assigned their fingerprint words, the  
 23 columns of the  $\Gamma$  code are permuted (at the chip level) as discussed above. An  
 24 object can now be fingerprinted with the fingerprinting words that are defined by  
 25 the permuted  $\Gamma$  code.

Fig. 4 shows a flow diagram that describes steps in an embedding method in accordance with the described embodiment. Step 100 builds or defines a suitable  $\Gamma$ -code, an exemplary one of which is shown in Fig. 3. Step 102 permutes the columns of the  $\Gamma$ -code in a manner that is known only to the embedder and the decoder that will ultimately decode the fingerprints. Permutation of the columns can take place by randomly shuffling the chips for all of the users (the same permutation for all the users). The permutation is the same for all of the users. An example of a suitable permutation was given above. Once the columns have been permuted, step 104 embeds a unique fingerprinting word in each of a number of different objects that are desired to be protected. An example of an embedding process is given just below. After the embedding process, the protected objects can be distributed.

Assume that a vector  $m=(m_1,...m_u)$  is defined that represents an object or signal that is to be protected. A spread sequence  $x = (x_1,...x_u)$  is to be used as an embedded spread sequence. Here,  $(\forall j)[x_j \in \{+1, -1\}]$ , and the signal is over a large alphabet whose size is not important for this discussion. When the object is embedded with a data symbol  $+D$  (or  $-D$ ), the resultant marked signal is designated as  $b = \{b_1...b_k\}$ , where  $(\forall j)[b_j = m_j + (-) x_j]$ .

Assume also that an adversary attempts to jam the protected object signal by adding a noise element  $J_i$  to each component, where  $J_i$  is at the same energy level as the spread sequence, i.e.  $J_i \in \{+1, -1\}$ , but it is uncorrelated with the spread sequence. After the jamming attack the signal can be represented as  $a=(a_1...a_u)$ , where  $(\forall j)[a_j = m_j + x_j + J_j]$ . Accordingly, the vector  $a$  represents the protected object as seen by the detector, (i.e. after embedding and after jamming attacks).

## Chip Detection

A first step in the detection process when an object is received is to unpermute the columns that were previously permuted. Recall that after the fingerprinting words are assigned but before an object is embedded, the columns (at the chip level) of the  $\Gamma$ -code are randomly permuted. Both the embedder and the detector know the random permutation. After the columns are unpermuted, the chips are detected in the received object. In this example, the received object is represented as  $a=(a_1...a_u)$  and the chips are detected by comparing the received object with an original expected object  $m=(m_1,...m_u)$ . Each component, e.g. pixel,  $a_i$  is compared with the expected unfingerprinted component, e.g. pixel,  $m_i$ . The following table lists the comparisons and their outcomes: We use  $z'_i$  to denote the detected chip  $i$ . This may differ from the original chip  $x_i$ , due to attacks.

Comparison	Outcome
$a_i > m_i$	Chip $z'_i = +1$
$a_i < m_i$	Chip $z'_i = -1$
$a_i = m_i$	Chip $z'_i = 0$

With the individual chips having been identified, attention is now turned to detecting a user that likely constitutes a colluder.

## Clipping

In the described embodiment, each block in a fingerprinting word comprises  $d$  chips. These chips were previously detected as described above.

1 With the chips having been detected, the blocks that comprise the fingerprinting  
2 word are initially “clipped” in an effort to distinguish between so-called “seen”  
3 and “unseen” blocks. Recall that “seen” blocks are those blocks that can be  
4 ascertained by two or more users or entities because of their differences.  
5 Alternately, “unseen” blocks are those blocks that cannot be “seen” by users  
6 because they are identical. Hence, clipping the blocks as described below  
7 distinguishes the “seen” and “unseen” blocks.

8 In the discussion that follows, the analysis deals with blocks,  $\Gamma$ -symbols,  
9 and Error correcting codes over an alphabet whose symbols are the  $\Gamma$ -symbols. In  
10 a first step, a function is defined from which a relative weight can be calculated.  
11 The function is defined as follows:

12 Let  $x \in \{1, -1\}$  and  $y \in \{0, 1, -1\}$ . Define the function:

$$13 \quad f(y, x) = \begin{cases} 1 & \text{if } x \text{ is not equal to } y, \text{ and } y \text{ is not equal to } 0, \\ 0 & \text{Otherwise.} \end{cases}$$

14  
15 Let  $X = (x_1, \dots, x_d)$ , where  $x_i \in \{1, -1\}$  and  $Y = (y_1, \dots, y_d)$ , where  $y_i \in \{1, -1, 0\}$ . The weight of  $Y$  relative to  $X$  is  $w(Y, X)$ —which is the sum from  $i=1$  to  $d$  of  $f(y_i, x_i)$ . When the reference point,  $X$ , is known from the context, we omit it and write  $w(Y)$ .  
16  
17

18 It follows that when an original block  $i$  has a value  $C_i$ ’ (“light blocks”), then  
19 its weight relative to  $C_i$ ’ is zero. This holds true even after jamming. On the other  
20 hand, if the original block was  $C_i$  (“heavy block”), then its weight relative to  $C_i$ ’  
21 after maximal jamming has a mean  $d/2$ , with deviation  $O((d)^{1/2})$ . This means that  
22 the working range is *roughly*  $d/2$ .

23 With the above function having been defined, weight assignment and  
24 clipping steps can now take place. In the described embodiment, this takes place  
25 by receiving, as input, the detected chips  $z_i$  arranged as blocks of  $d$  chips each ( $B_1$ ,



$B_2, \dots$ ). The output of the weight assignment and clipping steps is the relative weight of each block, with blocks that are likely “unseen” being clipped to their working range value. This can be represented mathematically as follows:

**Input:** Detected chips  $z = (z_1, z_2, \dots)$ , arranged as blocks of  $d$  chips each ( $B_1, B_2, \dots$ )

**Output:** For each block  $B_i$  output its relative weight,  $w_i = w(B_i, C_i')$ , clipping blocks that are likely unseen to their working range value.

**Method:** Define  $\mu = d/2$ , and let  $\delta$  be a parameter that is defined just below.

For each block  $B_i$  {  
    If  $w(B_i) > (1-\delta)\mu$ , then set  $w_i = (1-\delta)\mu$ ;  
    Else, set  $w_i = w(B_i, C_i')$ ;  
}

}

**Parameter choice:**

For  $N$  users, assuming we want to defend against a collusion of size  $c$ , with error probability  $\epsilon$ , then we choose:

- Number of  $\Gamma$ -symbols per a fingerprint word  $= L = 2c \ln(2N/\epsilon)$ ,
- Block size  $= d = 8c^2 \ln(8cL/\epsilon)$ ,
- $f = 2 \ln(4c^2 \ln(2N/\epsilon)/\epsilon)$ ,
- $\delta = f/\sqrt{(d/2)}$ ,
- $\mu = d/2$ .

Fig. 5 shows a flow diagram that describes steps in a weight assignment and clipping method in accordance with the described embodiment, an example of which is given directly above. Step 200 gets the first block that is present in a fingerprinting word. Step 202 calculates the weight of the first block. In the described embodiment, the weight of a given block is calculated as set forth above. Step 204 determines whether the block is likely an “unseen” block and if

1 so, step 206 clips the block's weight to its working range value. If the block is  
2 likely "seen", then its weight is as calculated above (step 208). Step 210  
3 determines whether there are any additional blocks. If so, the method branches  
4 back to step 202. Step 212 determines whether there are any additional gamma  
5 symbols. If there are, the method returns to step 200. If there are not, the method  
6 quits.

### 7 8 **Detection of a Subset that Produced an Altered Object $x$**

9 With the weights having been calculated for the various blocks of the  
10 altered fingerprinting word, and with the working range having been defined as set  
11 forth above, attention is now turned to ascertaining a subset of the coalition that  
12 produced an altered object  $x$ . The method that is utilized to ascertain such a  
13 coalition is similar, in some respects, to the method of the BS-system discussed  
14 above. Primary differences lie in the use of the newly-defined weights for the  
15 blocks, as well as the use of the new working range.

### 16 17 **Algorithm 3**

18 Given  $x \in \{0,1\}^{dk}$ ,  $k=2c-1$ , find a subset of the coalition that produced  $x$   
19 (within a  $\Gamma$ -code blocks are numbered  $0, \dots, k-1$ , and "colors" are numbered  
20  $0, \dots, k$ ).

- 21 1. If  $w_0 > 0$  output "color 0 is guilty."
- 22 2. If  $w_{k-1} < d/2 - (fd)^{1/2}$  output "color  $k$  is guilty."
- 23 3. For all  $s=2$  to  $k-2$  do:
  - 24 a. Let  $K = w(x|R_s)$  (here the reference point for weight computation  
25 is  $(C'_{s-1}, C'_s)$ ).
  - b. If  $w_{s-1} < K/2 - ((K/2)\ln(2n/\epsilon))^{1/2}$ , then output "color  $s$  is guilty."

1       The approach discussed above is particularly useful in the context of using  
2 a  $\Gamma$ -code having a reduced size. Recall that in the BS-system, a  $\Gamma$ -code having a  
3 reduced size was defined when the size of the  $\Gamma$ -code was considered in light of  
4 the number of colluders that were to be defended against. In that example, each  
5 new row or color of the  $\Gamma$ -code defined a  $\Gamma$ -symbol, and multiple  $\Gamma$ -symbols were  
6 used to build fingerprinting words for all of the users. Each of the fingerprinting  
7 words were different and unique. The permuted forms of the fingerprinting words  
8 are used for embedding in an object to be protected. Each of the fingerprinting  
9 words, when unpermuted and analyzed in accordance with the BS-system's second  
10 algorithm yielded a user that likely constituted a colluder.

11       In the presently-described embodiment, a reduced-size  $\Gamma$ -code is also  
12 defined and includes a plurality of colors or rows. The number of colors or rows  
13 is a function of the number of colluders  $c$  that are desired to be defended against.  
14 That is, the number of colors or rows is defined, in this example, to be  $2c$ . Each  
15 color or row defines a  $\Gamma$ -symbol. The  $\Gamma$ -symbols that are being defined here are,  
16 however, very different from the  $\Gamma$ -symbols that are defined in the BS-system.  
17 Specifically, the presently-described  $\Gamma$ -symbols that make up the  $\Gamma$ -code each  
18 contain spread sequences, rather than collections of bits. In the specifically-  
19 discussed example, a fingerprinting word is composed of  $L$   $\Gamma$ -symbols, where a  $\Gamma$ -  
20 symbol is composed of  $2c-1$  blocks. A block, in turn, is composed of  $d$  chips,  
21 where a chip is a spread spectrum chip. Given this relationship, the size of a  
22 vector that represents the protected object is  $2dcL$ .

23       An exemplary, reduced-size  $\Gamma$ -code is shown in the table immediately  
24 below:  
25

Color	$\Gamma$ symbol
1	$\Gamma_1$
2	$\Gamma_2$
3	$\Gamma_3$
4	$\Gamma_4$
5	$\Gamma_5$
6	$\Gamma_6$

Here, there are six colors that define the  $\Gamma$ -code. These individual colors are used as the alphabet to build fingerprinting words for all of the users in the particular user universe. After the  $\Gamma$ -code is defined, each user or entity is assigned a fingerprinting word having  $L$  of these  $\Gamma$ -symbols, where  $L$  is a number that is selected so that no two users or entities have the same fingerprinting word. It also controls the error probability. With  $N$  users, and error probability  $\epsilon$  we need  $L=2c \cdot \log(2N/\epsilon)$ . This fingerprinting word serves to identify a user or entity later when an altered object is received. After the fingerprinting words are assigned, the columns are randomly permuted in a manner that is known to both the embedder and the detector. After permutation of the columns, individual objects that are desired to be protected are embedded with a permuted fingerprinting word that uniquely serves to identify an associated user or entity.

Recall that the way that protected objects typically get altered is that different entities or users get together and compare their protected objects. The concept of “seen” and “unseen” blocks was discussed above and refers, respectively, to blocks that have differences that can be ascertained by different colluders, and blocks that do not have differences and that cannot be seen by

1 colluders. In accordance with the marking assumption discussed above, it is  
2 assumed, logically, that colluders will manipulate or adjust only the blocks that  
3 they can see. Accordingly, “unseen” blocks will not be manipulated or adjusted  
4 by colluders. Thus, when an altered object is received, it has a fingerprinting word  
5 that has been manipulated by two or more colluders. It may also be the case that  
6 random jamming may occur on the unseen bits.

### 7 8 **Detection of an Entity that Likely Constitutes a Colluder**

9 The manipulated or altered fingerprinting word contains  $L$   $\Gamma$ -symbols. In  
10 the described embodiment, each of the individual constituent  $\Gamma$ -symbols in the  
11 altered fingerprinting word is analyzed and a set of one or more likely colors that  
12 might be the subject of a collusion is built. When all of the  $\Gamma$ -symbols in the  
13 altered fingerprinting word have been analyzed in this manner, an  $m \times L$ , (where  $m$   
14 is the number of  $\Gamma$ -symbols or colors, i.e.  $m=2c$ ) matrix is defined that contains an  
15 indication of which colors might be the subject of a collusion for each of the  $\Gamma$ -  
16 symbols in the altered fingerprinting word. The fingerprinting word for each of  
17 the users or entities is then compared with the matrix. Specifically, each  $\Gamma$ -symbol  
18 of the user’s fingerprinting word is compared with the set of likely colors for the  
19 corresponding  $\Gamma$ -symbol of the altered fingerprinting word. If the user’s  $\Gamma$ -symbol  
20 coincides with one of the colors in the set, then a counter is incremented. If there  
21 is no coincidence, then the counter is not incremented and the next  $\Gamma$ -symbol for  
22 the user is checked. This process continues until all of the  $\Gamma$ -symbols for all of the  
23 users have been checked. At this point in the process, all of the users will have a  
24 value associated with their counter. The most likely colluder is the user that has  
25 the highest counter value.

Fig. 6 shows a flow diagram that describes steps in a detection method in accordance with the described embodiment. Step 300 receives a protected object that has a fingerprinting word that has been altered by a user or entity. Step 302 unpermutes the columns (at the chip level) of the altered fingerprinting word. Step 304 evaluates each of the  $\Gamma$ -symbols in the altered fingerprinting word. In the described embodiment, each of the  $\Gamma$ -symbols is evaluated by applying Algorithm 3 (above) to the  $\Gamma$ -symbol. Application of Algorithm 3 produces a matrix (step 306) of likely colors that might be the subject of a collusion. Production of the described matrix takes place by selecting a  $\Gamma$ -symbol if the weight of a block satisfies a predefined relationship that is specified, in this example, by Algorithm 3. Step 308 then gets the first user's fingerprinting word and step 310 evaluates the user's fingerprinting word by comparing the first  $\Gamma$ -symbol in the user's fingerprinting word with a set of one or more colors from the matrix. In the described embodiment, the matrix has L columns, each of which corresponds to a different  $\Gamma$ -symbol of a fingerprinting word. For any one column, there is a set of one or more colors that are produced by Algorithm 3. Each of the produced colors in a column are used for comparison with a corresponding  $\Gamma$ -symbol in a user's fingerprinting word. This will become more apparent in the example that is given below. Step 312 determines whether the user's particular fingerprinting word  $\Gamma$ -symbol coincides with one of the colors in the set of colors for the corresponding column in the matrix. If there is a coincidence, then step 314 increments the user's counter. If there is not a coincidence, then step 316 determines whether there are any additional  $\Gamma$ -symbols for the user. If there are, then step 318 gets the next  $\Gamma$ -symbol and loops back to step 310. If there are no additional  $\Gamma$ -symbols for the user, then step 320 determines whether there are any additional users. If there are

1 additional users, then the method loops back to step 308 and gets the new user's  
2 fingerprinting word. If there are no additional users, then step 322 selects the user  
3 with the highest counter value and incriminates them as a colluder.

4 As an example to assist in understanding the above-described process,  
5 consider the following elementary example using the following  $\Gamma$ -code:

Color	$\Gamma$ symbol
1	$\Gamma_1$
2	$\Gamma_2$
3	$\Gamma_3$
4	$\Gamma_4$
5	$\Gamma_5$
6	$\Gamma_6$

15 Assume that each fingerprinting word has a length  $L$  that, in this example,  
16 is five  $\Gamma$ -symbols long. Applying Algorithm 3 to each of the five  $\Gamma$ -symbols  
17 might yield the following matrix:

### Matrix

Color	Implicated Color $\Gamma_1$	Implicated Color $\Gamma_2$	Implicated Color $\Gamma_3$	Implicated Color $\Gamma_4$	Implicated Color $\Gamma_5$
1		X	X		
2	X				
3	X		X	X	
4					X
5		X			X
6			X		X

Here, each of the last five columns corresponds to an individual  $\Gamma$ -symbol in the altered fingerprinting word and contains a number of “X” marks. Each “X” indicates, for a particular  $\Gamma$ -symbol, a color that might be the subject of a collusion. Each  $\Gamma$ -symbol in the altered fingerprinting word has a set of one or more colors associated with it. In this example, for the first  $\Gamma$ -symbol in the altered fingerprinting word, colors 2 and 3 might be the subject of the collusion. For the second  $\Gamma$ -symbol in the fingerprinting word, colors 1 and 5 might be the subject of the collusion, and so on. After this matrix is defined, each user’s fingerprinting word is compared,  $\Gamma$ -symbol by  $\Gamma$ -symbol, with the implicated colors for each of the corresponding  $\Gamma$ -symbols in the matrix. This comparison is summarized in the table that appears below:



User 1 Fingerprinting word	1	1	4	6	5
Counter 1	0	1	1	1	2
User 2 Fingerprinting word	2	5	3	3	4
Counter 2	1	2	3	4	5

Here, there are two hypothetical users designated user 1 and user 2. Each user has a unique fingerprinting word that is represented numerically by its constituent colors. For example, the fingerprinting word for user 1 is as follows [(color 1) (color1) (color 4) (color 6) (color 5)]. This can also be represented as ( $\Gamma_1$   $\Gamma_1$   $\Gamma_4$   $\Gamma_6$   $\Gamma_5$ ). To determine which of the two users is incriminated in this example, each of the user's  $\Gamma$ -symbols or colors is checked against the corresponding incriminated colors for the corresponding  $\Gamma$ -symbol in the matrix above. If the user's  $\Gamma$ -symbol is found in the matrix, then the user's counter is incremented for that  $\Gamma$ -symbol. Thus, for user 1, its first  $\Gamma$ -symbol is defined by color 1. Reference to the matrix indicates that, for the first  $\Gamma$ -symbol, color 1 is not incriminated. Accordingly, the user's counter is not incremented. For user 1's second  $\Gamma$ -symbol, (defined by color 1) however, color 1 is among the set of colors that are implicated for the second  $\Gamma$ -symbol of the altered fingerprinting word. Accordingly the counter is incremented by one. Similar analysis continues for each of the remaining  $\Gamma$ -symbols, and for each of the remaining users. After all of the users have been checked against the matrix, the user with the highest counter value (right most counter column) is selected as a colluder. In this example, user 2 has the higher of the counter values because there are more coincidences between its fingerprinting word and the incriminated colors of the matrix.

1       The methods and systems described above can greatly increase the number  
2 of colluders that can be defended against over the number enabled by the Boneh-  
3 Shaw system. For example, assume that a movie has around  $10^{10}$  pixels and that  
4 10% of the pixels are significant enough so that data can be hidden in them. This  
5 means that  $10^9$  chips can be utilized in connection with this movie. Assuming that  
6 there are  $N=10^6$  users and an error rate of  $10^{-3}$  is desired, then the number of  
7 colluders that can be defended against is  $c=78$ . Note that with the above  
8 parameters we still may accuse about 1000 entities, where there are only 78  
9 colluders. Hence accusations should take place only with those repeatedly  
10 incriminated. However, the number 78 compares favorably with  $c=4$  for Boneh-  
11 Shaw. Being able to defend against more colluders increases the breadth of  
12 protection and desirably makes it much more difficult for fingerprinting words to  
13 be altered. The required value of parameter  $d$  is  $d=2c^2 * \log(8cL/\epsilon)$ .

14       In compliance with the statute, the invention has been described in  
15 language more or less specific as to structural and methodical features. It is to be  
16 understood, however, that the invention is not limited to the specific features  
17 described, since the means herein disclosed comprise preferred forms of putting  
18 the invention into effect. The invention is, therefore, claimed in any of its forms  
19 or modifications within the proper scope of the appended claims appropriately  
20 interpreted in accordance with the doctrine of equivalents.  
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22  
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25